

Gauged $U(1)_R$ supergravity on orbifold ¹

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Abstract. We discuss a gauged $U(1)_R$ supergravity on five-dimensional orbifold (S^1/Z_2) in which a Z_2 -even $U(1)$ gauge field takes part in the $U(1)_R$ gauging, and show the structure of Fayet-Iliopoulos (FI) terms allowed in such model. Some physical consequences of the FI terms are examined.

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INTRODUCTION

Recently five-dimensional (5D) supergravity (SUGRA) on the orbifold S^1/Z_2 has been studied as an interesting theoretical framework for physics beyond the SM. It has been noted that 5D orbifold SUGRA with a $U(1)_R$ symmetry gauged by the Z_2 -odd graviphoton can provide the supersymmetric Randall-Sundrum (RS) model [1] in which the weak to Planck scale hierarchy can arise naturally from the geometric localization of 4D graviton [2], and/or Yukawa hierarchy can be generated by the quasi-localization of the matter zero modes in extra dimension where we generically have an interesting correlation between the flavor structure in the sparticle spectra and the hierarchical Yukawa couplings [3]. In the former case, the bulk cosmological constant and brane tensions which are required to generate the necessary AdS_5 geometry appear in the Lagrangian as a consequence of the $U(1)_R$ FI term with Z_2 -odd coefficient.

In this talk we consider a more generic orbifold SUGRA which contains a Z_2 -even 5D gauge field A_μ^X participating in the $U(1)_R$ gauging [4]. If 4D $N = 1$ SUSY is preserved by the compactification, the 4D effective theory of such model will contain a gauged $U(1)_R$ symmetry associated with the zero mode of A_μ^X , which is not the case when the 5D $U(1)_R$ is gauged only through the Z_2 -odd graviphoton. Based on the known off-shell formulation [5], we formulate a gauged $U(1)_R$ SUGRA on S^1/Z_2 in which both A_μ^X and the graviphoton take part in the $U(1)_R$ gauging and then analyze the structure of FI terms allowed in such model. As expected, introducing a Z_2 -even $U(1)_R$ gauge field accompanies new bulk and boundary FI terms in addition to the known integrable boundary FI term which could be present in the

absence of any gauged $U(1)_R$ symmetry [6]. As we will see, those new FI terms can have interesting implications to the quasi-localization of the matter zero modes in extra dimension and the SUSY breaking [3] and also to the radion stabilization.

FORMULATION

For a minimal setup, we introduce two vector multiplets and two hypermultiplets in the off-shell formulation of 5D (conformal) SUGRA [5]:

$$\begin{aligned}\mathcal{V}_Z &= (M^Z = \alpha, A_\mu^Z, \Omega^{Zi}, Y^{Zij}), \\ \mathcal{V}_X &= (M^X = \beta, A_\mu^X, \Omega^{Xi}, Y^{Xij}),\end{aligned}$$

and

$$\begin{aligned}\mathcal{H}_c &= (\mathcal{A}_i^x, \eta^x, \mathcal{F}_i^x), \\ \mathcal{H}_p &= (\Phi_i^x, \zeta^x, F_i^x),\end{aligned}$$

with the norm function

$$\mathcal{N} = \alpha^3 - \frac{1}{2}\alpha\beta^2,$$

and the hypermultiplet gauging

$$\begin{aligned}(t_Z, t_X)\Phi &= (c\varepsilon(y), q)i\sigma_3\Phi, \\ (t_Z, t_X)\mathcal{A} &= \left(-\frac{3}{2}k\varepsilon(y), -r\right)i\sigma_3\mathcal{A},\end{aligned}$$

where we adopt the 2×2 matrix notations omitting $x = 1, 2$ index and $SU(2)$ indices $i, j = 1, 2$, and the hyper-scalars satisfy the reality condition $\mathcal{A}^* = i\sigma_2\mathcal{A}i\sigma_2^T$, $\Phi^* = i\sigma_2\Phi i\sigma_2^T$. The Z_2 -even bosonic (non-auxiliary) components are $\alpha, A_\mu^Z, A_\mu^X, \mathcal{A}_{i=2}^{x=2}$ and $\Phi_{i=2}^{x=2}$, and $\mathcal{V}_Z, \mathcal{H}_c$ are the graviphoton vector multiplet and the compensator hypermultiplet respectively. The Z_2 -odd coefficient $\varepsilon(y)$ in the hypermultiplet gauging is consistently introduced by the mechanism proposed in [7]. The nonzero value of

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the charge r corresponds to the $U(1)_R$ symmetry gauged by Z_2 -even vector field A_μ^X .

The bosonic part of the Lagrangian is given by

$$\begin{aligned}\mathcal{L}_{\text{bosonic}} &= \mathcal{L}_{\text{bulk}} + \mathcal{L}_{\partial\mathcal{E}} + \mathcal{L}_{N=1}, \\ e^{-1}\mathcal{L}_{\text{bulk}} &= -\frac{1}{2}R - \frac{1}{4}a_{IJ}F_{\mu\nu}^I F^{\mu\nu J} + \frac{1}{2}a_{IJ}\nabla^m M^I \nabla_m M^J \\ &\quad + \frac{1}{8}e^{-1}C_{IJK}\varepsilon^{\lambda\mu\nu\rho\sigma}A_\lambda^I F_{\mu\nu}^J F_{\rho\sigma}^K \\ &\quad + \text{tr}\left[|\nabla_m\Phi|^2 - |\nabla_m\mathcal{A}|^2 - |V_m|^2\right. \\ &\quad \left.- M^I M^J (\Phi^\dagger t_I^\dagger t_J \Phi - \mathcal{A}^\dagger t_I^\dagger t_J \mathcal{A})\right] \\ &\quad - \frac{1}{2}\text{tr}\left[\mathcal{N}_{IJ}Y^{I\dagger}Y^J - 4Y^{I\dagger}(\mathcal{A}^\dagger t_I \mathcal{A} - \Phi^\dagger t_I \Phi)\right], \\ e_{(4)}^{-1}\mathcal{L}_{\partial\mathcal{E}} &= -2\alpha(3k + \frac{3}{2}k\text{tr}[\Phi^\dagger\Phi] + c\text{tr}[\Phi^\dagger\sigma_3\Phi\sigma_3]) \\ &\quad \times (\delta(y) - \delta(y - \pi R)), \\ e_{(4)}^{-1}\mathcal{L}_{N=1} &= M_{(4)}^2 \left[-2r\left(2Y^{X(3)} - e^{-1}e_{(4)}\partial_y\beta\right) - \frac{1}{2}R^{(4)}\right] \\ &\quad \times (\Lambda_0\delta(y) + \Lambda_\pi\delta(y - \pi R)),\end{aligned}$$

where the matrix notations are employed again, $I, J = (Z, X)$, $a_{IJ} = -\frac{1}{2}\frac{\partial^2 \ln \mathcal{N}}{\partial M^I \partial M^J}$, $M_{(4)}^2 = (1 + \frac{1}{2}\text{tr}[\Phi^\dagger\Phi])^{2/3}$ and $V_m = \frac{1}{2}(\Phi^\dagger(\nabla_m\Phi) - (\nabla_m\Phi)^\dagger\Phi) - \frac{1}{2}(\mathcal{A}^\dagger(\nabla_m\mathcal{A}) - (\nabla_m\mathcal{A})^\dagger\mathcal{A})$. Here we have included only 4D $N=1$ pure SUGRA action at the orbifold fixed points without any Kähler and superpotentials for simplicity. We remark that after the superconformal gauge fixing,

$$\mathcal{N} = 1, \quad \mathcal{A} = \mathbf{1}_2 \sqrt{1 + \text{tr}[\Phi^\dagger\Phi]/2},$$

we find the bulk FI term $-e(6kY^{Z(3)} + 4rY^{X(3)})$ in $\mathcal{L}_{\text{bulk}}$ and the boundary FI term $-2re_{(4)}M_{(4)}^2(2Y^{X(3)} - e^{-1}e_{(4)}\partial_y\beta)(\Lambda_0\delta(y) + \Lambda_\pi\delta(y - \pi R))$ in $\mathcal{L}_{N=1}$ for the auxiliary fields $Y^{Z,X}$ in the vector multiplets.

We are interested in the 4D Poincaré invariant background geometry,

$$ds^2 = e^{2K(y)}\eta_{\underline{\mu}\underline{\nu}}dx^\underline{\mu}dx^\underline{\nu} - dy^2,$$

and the gravitino-, hyperino- and gaugino-Killing parameters on this background are given respectively by

$$\begin{aligned}\kappa &= \partial_y K - \mathcal{P}/3 \\ F &= \partial_y v - (q\beta + c\varepsilon(y)\alpha - \mathcal{P}/2)v \\ D &= \partial_y \phi + g^{\phi\phi}\mathcal{P}_\phi \\ &\quad - 2rM_{(4)}^2 g^{\phi\phi}\beta_\phi(\Lambda_0\delta(y) - \Lambda_\pi\delta(y - \pi R)),\end{aligned}$$

where

$$\begin{aligned}\mathcal{P} &= -2\left[\frac{3}{2}k\varepsilon(y)\alpha + r\beta\right. \\ &\quad \left.+ \left\{\left(\frac{3}{2}k + c\right)\varepsilon(y)\alpha + (r+q)\beta\right\}v^2\right],\end{aligned}$$

and ϕ is the physical gauge scalar field parameterizing the (very special) manifold of vector multiplet determined by $\mathcal{N} = \alpha^3(\phi) - \alpha(\phi)\beta^2(\phi)/2 = 1$ with the metric $g_{\phi\phi} = a_{IJ}M_\phi^I M_\phi^J$. We choose $\alpha(\phi) = \cosh^{2/3}(\phi)$ and $\beta(\phi) = \sqrt{2}\cosh^{2/3}(\phi)\tanh(\phi)$ in the following. The real and diagonal component of the quaternionic hyperscalar field Φ is represented by v in the Killing parameters, and zero vacuum values are assumed for the other components for simplicity. In terms of these Killing parameters, the 4D energy density is found to be

$$E = \int dy e^{4K} \left(\frac{1}{2}g_{\phi\phi}D^2 + \frac{2}{1+v^2}|F|^2 - 6|\kappa|^2 \right),$$

and it is obvious that the Killing condition $\kappa = D = F = 0$ determines a stationary point of the 4D scalar potential if the solution exists.

PHYSICAL CONSEQUENCES

Now we examine some physical consequences of the 5D gauged $U(1)_R$ supergravity on S^1/Z_2 which can have the bulk and the boundary FI term, for the supersymmetric vacuum configurations, $\kappa = D = F = 0$.

First we consider the case that we have a charged hypermultiplet Φ with the charge satisfying $q/r < -1$. For $k = c = 0$ that results in $K(y) \simeq 0$, the vacuum values of the scalar fields are given by

$$\phi = 0, \quad v = v_0 \equiv \pm \sqrt{-\frac{r}{r+q}},$$

for $\Lambda_{0,\pi} = 0$, and

$$\begin{aligned}\phi &\simeq -2\frac{|rv_0|}{rv_0}\sqrt{1+\frac{r}{q}}(A_+e^{\omega y} - A_-e^{-\omega y}), \\ v &\simeq v_0 + (A_+e^{\omega y} + A_-e^{-\omega y}),\end{aligned}$$

for $\Lambda_{0,\pi} \neq 0$, where

$$A_\pm = \pm \frac{\sqrt{2}|rv_0|}{2v_0} \left(\frac{q}{r+q} \right)^{7/6} \frac{\Lambda_0 + \Lambda_\pi e^{\pm\omega\pi R}}{e^{\pm 2\omega\pi R} - 1},$$

and $\omega = \sqrt{-8rq}$. We find a nontrivial y -dependent vacuum values for the latter case due to the boundary FI term. Notice that the vacuum value of the gauge scalar $\phi(y)$ gives the y -dependent mass for the charged hypermultiplets which results in nontrivial zero-mode wavefunctions for them. We will show the zero-mode profile in the next more simple but interesting case.

Next we consider the case there are charged chiral multiplets $Z_{0,\pi}$ with the charge $q_{0,\pi}^Z$ at the orbifold fixed points $y = 0, \pi R$ respectively, but no hypermultiplets with the charge $q/r < -1$ in bulk. We introduce minimal Kähler potential and no superpotential for them at the

fixed points. For $k = c = 0$, the vacuum values of the scalar fields are given by

$$\phi = 2\sqrt{2}ry + \sqrt{2}\lambda_0, \quad v = 0,$$

where $\lambda_{0,\pi} = (r + \sum_z q_{0,\pi}^z |z_{0,\pi}|^2) \Lambda_{0,\pi}$ and the orbifold radius is determined by $2\pi R = -\frac{\lambda_0 + \lambda_\pi}{r}$. We find a linear profile of ϕ in the y -direction due to the bulk FI term, which results in the Gaussian form of the zero-mode wavefunction for the charged hypermultiplet,

$$\Phi^{(0)}(y) \simeq \Phi^{(0)}(0) e^{2(q+r)(ry^2 + \lambda_0 y)}.$$

The ratio of the wavefunction values between two fixed points are then shown to be $\frac{\Phi^{(0)}(0)}{\Phi^{(0)}(\pi R)} \approx e^{-\frac{q+r}{2r}(\lambda_\pi^2 - \lambda_0^2)}$. Some numerical plots are shown in Fig. 1 for $c \neq 0$ but $k = 0$ and in Fig. 2 for both $c, k \neq 0$. From these figures we find that the nonvanishing r (i.e., gauging $U(1)_R$ by Z_2 -even vector field) as well as the bare kink mass c affects the zero-mode profiles of the charged hypermultiplets significantly. The nonvanishing charge k changes the linear profile of ϕ resulting in a more/less severe localization of the charged hypermultiplet zero-mode, depending on the sign of kr .

SUMMARY

We have studied a 5D gauged $U(1)_R$ supergravity on S^1/Z_2 in which both a Z_2 -even $U(1)$ gauge field and the Z_2 -odd graviphoton take part in the $U(1)_R$ gauging. Based on the off-shell 5D supergravity of Ref. [5], we examined the structure of Fayet-Iliopoulos (FI) terms allowed by such theory. As expected, introducing a Z_2 -even $U(1)_R$ gauging accompanies new bulk and boundary FI terms in addition to the known integrable boundary FI term which could be present in the absence of any gauged $U(1)_R$ symmetry. The new (non-integrable) boundary FI terms originate from the $N = 1$ boundary supergravity, and thus are free from the bulk supergravity structure in contrast to the integrable boundary FI term which is determined by the bulk structure of 5D supergravity [6].

We have examined some physical consequences of the Z_2 -even $U(1)_R$ gauging in several simple cases. It is noted that the FI terms of gauged Z_2 -even $U(1)_R$ can lead to an interesting deformation of vacuum structure which can affect the quasi-localization of the matter zero modes in extra dimension and also the SUSY breaking and radion stabilization. Thus the 5D gauged $U(1)_R$ supergravity on orbifold has a rich theoretical structure which may be useful for understanding some problems in particle physics such as the Yukawa hierarchy and/or the supersymmetry breaking [3]. For such phenomenological study and for the analysis of the radion stabilization,

the $N = 1$ superfield description [8] will be useful. When one tries to construct a realistic particle physics model within gauged $U(1)_R$ supergravity, one of the most severe constraint will come from the anomaly cancellation condition. In some cases the Green-Schwarz mechanism might be necessary to cancel the anomaly, which may introduce another type of FI term into the theory [9]. These issues will be studied in future works.

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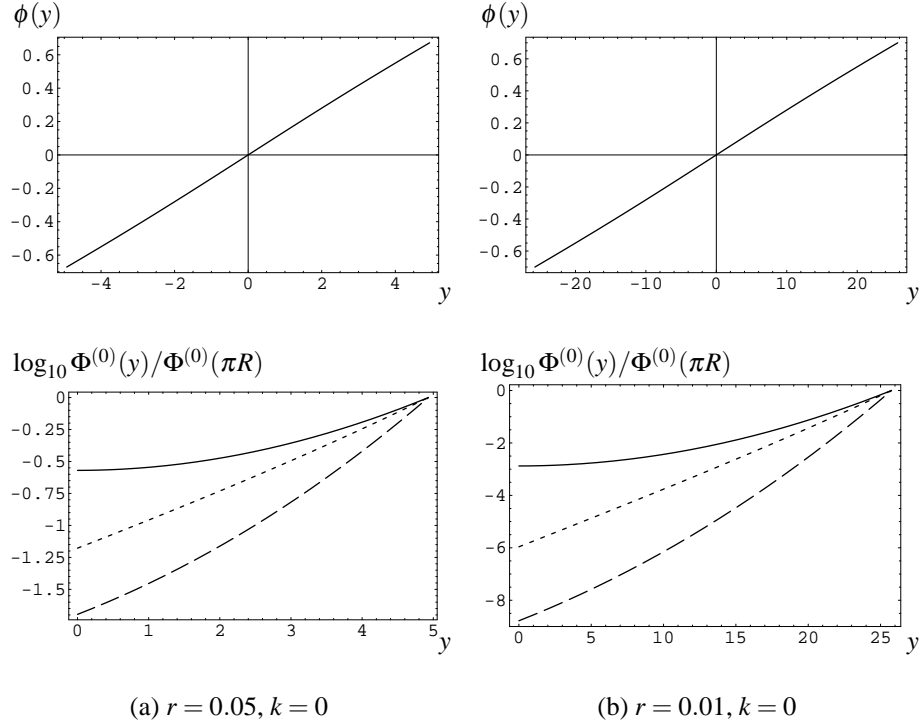


FIGURE 1. The profiles of ϕ and the matter zero mode $\Phi^{(0)}$ for some cases with $k = 0$ and $\lambda_0 = 0$. Here we choose $\lambda_\pi = (r-1)/2$. For the matter zero mode profile, the solid-, dotted- and dashed-curves represent the case with $(q, c) = (0.5, 0)$, $(0, 0.5)$ and $(0.5, 0.5)$, respectively. All the curves are shown within $|y| \leq \pi R$.

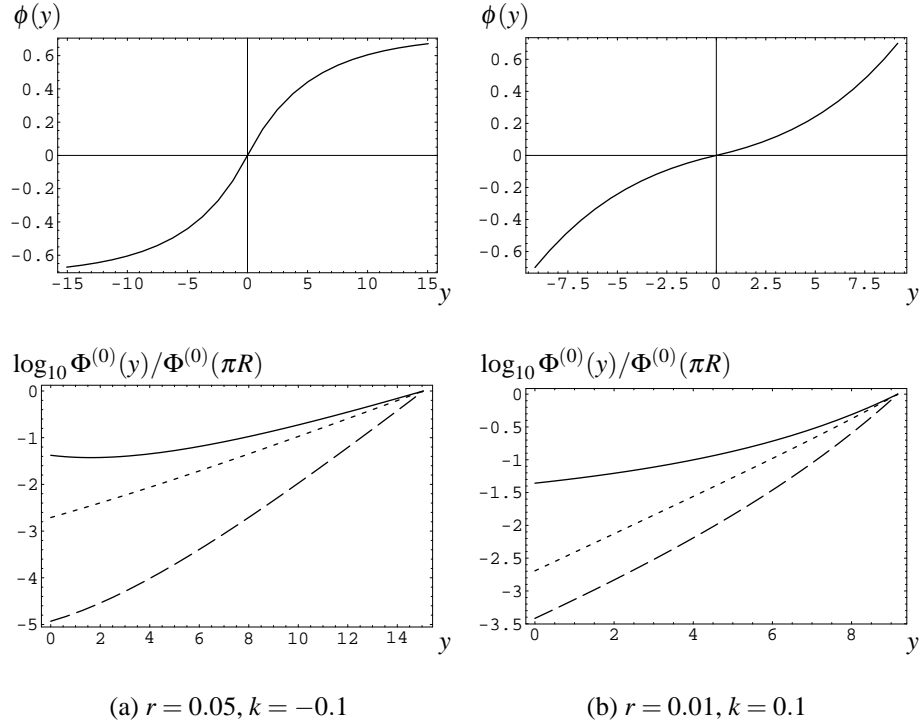


FIGURE 2. The profiles of ϕ and $\Phi^{(0)}$ for $r, k \neq 0$, $\lambda_0 = 0$ and $\lambda_\pi = (r-1)/2$. Again the solid-, dotted- and dashed-curves represent the case $(q, c) = (0.5, 0)$, $(0, 0.5)$ and $(0.5, 0.5)$, respectively. Note that $K \simeq -ky$ in this supersymmetric solution.